

Sequenced Teaching of Problem Solving

HOME LEARNING MATERIAL

© STOPSPROBLENSOLVING UK 2020



Introduction

The modern maths curriculum in schools places a great focus on children's ability to solve problems and reason mathematically. When learning maths, children must be able to apply the core skills they have learned to a variety of problems and challenges.

At STOPS, we have devised 8 key problem-solving strategies that will help children approach problems with confidence. For each strategy, we have a range of problems that increase in difficulty so that children learn to tackle any tough maths problems with confidence.

Your child uses the STOPS problems in school and this book is designed to support and supplement the work that they are doing in school.

The problems and guidance in this book are designed to run in parallel to the work that your child is doing in school, enabling you to support them to become great mathematicians and successful problem-solvers - ensuring them success all the way to the end of primary school.

The STOPS Problem Solving Strategies:

A. Act it out / make a model

A great way to start solving problems. Act out, make or draw what the problem shows and you will be well on the way to solving it.

B. Trial and Error

A strategy every child must have - simply make some guesses and see how they go. Much better than not knowing how to start.

C. Trial By Improvement

The next step. Make an estimate, get a solution. Is it correct? Why not? How can we change our estimate to improve it? Children become more systematic.

D. Make a list or table

Many problems can be tackled by making a list of potential solutions. This can go hand-in-hand with strategies B and C to give children serious mental tools with which to solve tricky problems. Later, turn your list into organised tables and you can solve anything.

E. Find the pattern

Many problems can be solved by identifying a repeating pattern in shapes or numbers and using it to predict what may happen in other situations.

F. Simplify the problem

Some problems can be intimidating for children, but by making it more simple, it becomes more accessible.

G. Work Backwards

Start at the end and work back. Children will refine their skills of reasoning and 'inverse operations' to work their way through maths problems with ease.

H. Solve algebraically

It sounds more difficult than is, especially to children. When broken down into manageable steps of learning using shapes, symbols and eventually letters, children will become confident and experts in using algebra to solve problems

Using this booklet and how to best help your child

Each strategy has a one main problem to work through with your child and one other supporting problem. There are different steps within each strategy that make sure the problems are age-appropriate for your child. Remember that problem solving skills are very different to maths skills and children can develop at very different rates.

Each problem has notes afterwards that will give you guidance and examples of questions or modifications to support them if they are not sure or questions that could extend them if they are finding it easy.

Each problem is based on the STOPS problem-solving grid, where each strategy has 7 steps of difficulty for each of the 8 strategies. At the top of each problem in this book is the 'step' that the problem comes from, so that you can pick up on the next step of the strategy in every school year.

Some general tips:

- Encourage your child to make mistakes and feel positive about them, this is the only way to learn.
- Encourage children to record their thoughts in writing, on paper or in a special 'problem solving' notebook.
- Allow children time to think through for themselves, do not be tempted to do too much for them.

How do 'steps' and year groups work?

The STOPS problem solving skills are based on our famous grid, where each of the strategies has 7 steps within it that must be completed to be an expert problem-solver.

Each strategy is different, so 'step 1' does not always mean 'year 1'.

Below is our grid, with the recommended year 4 problems highlighted. This book will provide support and companion problems to the year 4 set of problems that your child is studying at school.



STRATEGY A -ACT IT OUT/MAKE A MODEL



Each line of shelters has a total of 21 people.

How many people live in each shelter?

- For a fun way to boost vocabulary, see if your child can name the shelters, e.g. igloo, caravan, tent, hut, wigwam.
- You can model the activity with buttons or counters on a grid, or just use pencil and paper.
- Encourage mental addition. A good strategy for adding three numbers is to look for number bonds. Eg, if you need to add 7, 9 and 3 you can ask the child if they can see any bonds to 10. 7 and 3 make 10 so adding the 9 is easier.
- If they find it too hard, give them a number or two to start with, from the solution.
- If they find it hard, ask them to find an alternative solution.

Solutions:

Different versions of this solution are possible. One solution is:

5 - 7 - 9 6 4 10 - 3 - 8



Can you arrange the numbers 1–8 into the boxes so that no consecutive numbers are joined by a line?

- Check that children know the vocabulary: consecutive means numbers that are next to each other when we count. 3 and 4 are consecutive, 5 and 7 are not.
- Allow children to make some experimental trials and re-enforce that making errors is part of the fun.
- Use pencil and paper so that errors can be rubbed out
- If children find it hard, tell them to put consecutive numbers at other ends of the shape
- If children find it easy, ask them to find different solutions how many different ways can they solve it?

Solutions:

Other solutions are possible.



STRATEGY B -TRIAL AND ERROR

© STOPSPROBLEMSOLVING UK 2020

Step 6 - Maths Maze



Start at 0. Can you follow the stops to find a path that makes exactly 100?

Which path makes the largest total?

Which path makes the smallest total?

- Encourage children to make jottings using a pencil and paper
- This is a great puzzle to practice quick mental arithmetic.
- Revise strategies for doubling, for example you can 'partition' the tens and units and double each. We can double 35 by doubling 30 and doubling 5, then adding these up.
- You can practice times tables before starting this problem. Children in year 4 should know all times tables up to 12x12, so why not use this puzzle to practise some tables, e.g. the 7 times table.
- Emphasise that this strategy is "Trial and Error', so don't let them be discouraged by mistakes.
- If children find it hard, get them started on the first few steps of the correct solution
- If children find it easy, ask them to start at 10 and get a total of 265 (+7, x5, +15, x5, ÷2, +15). You can make up your own of these by picking a number to start and creating a path before asking your child which path you followed.

Solutions:

To make 100: +7, x4, +12, x2, +5, +15

Largest total: +7, x5, +15, x5, +5, +15 (270)

Smallest total: +8, x3, ÷4, +10, ÷2, +15 (23)

Step 6 - Making 30

Can you complete this shape with the numbers 7-14 so that each line has a total of 30?



- Encourage children to make jottings using a pencil and paper
- Quick mental addition is the key here. Revise 'number bonds' to 10 (eg 7 + 3, 6 + 4) and use these to spot number bonds to 20 and then 30. For example, if I know 7+3 = 10, 17 + ? = 20 and 17 + ? = 30.
- Children can use written methods for adding if they need to but quick mental strategies are better
- If children find it hard, give them one or two starting numbers
- If children find it easy, ask them to find several solutions

Solutions:

Other solutions may be possible.



Step 7 - Screen Unlock

Swipe your finger in straight-line patterns to unlock the screen. You must start from 52 and finish on 115.



- Encourage children to make jottings using a pencil and paper
- Children may need column addition or multiplication for some of the trickier calculations.
- Ensure that children record their unsuccessful trials.
- If they find it easy, give them the first one or two moves
- If they find hard, they can make their own path for you to follow, with different start and end numbers!

Solution:



Step 7 - Blank number addition!

Can you fill in the numbers 1–9 to make this addition correct?



© STOPSPROBLEMSOLVING UK 2020

- First, make sure that your child is confident with column addition and is able to 'carry' or 're-group' into the next column when the total exceeds 9.
- Ensure you have a pencil and eraser
- You can begin with random trials or by using reasoning, for example "I know that 8 and 1 make 9, so maybe I can put them in a column"
- If you carry a 1 into the next column, this does not count as the digit 1; all digits 1-9 must be placed into the spaces.
- If children find it hard, fill in some blank spaces, for example put the bottom row in.
- If children find it easy, ask them how many different solutions they can find and what number patterns they notice in the answer.

Solution:



Many solutions are possible, for example: 219 + 438 = 657, or 273 + 546 = 819, or 327 + 654 = 981 or 192 + 284 = 576. One pattern that might be noticed is that all of the 3-digit answers add up to 18.

STRATEGY C TRIAL BY IMPROVEMENT

Step 5 - Sticker Collection

Aisha and Roman have collected some animal stickers. They each have the same number of stickers. They stick their stickers into an album, where each full sheet has the same number of stickers.



Josh says, "I have 5 full sheets and 2 loose stickers."

Roman says "I have 3 full sheets and 12 loose stickers."

How many stickers are on a full sheet?

- First, ensure that your child understands the problem. You could make your own sticker books if you have the time!
- The strategy is 'trial by improvement', so we can pick random numbers and see if they work.
- Start with a trial of, for example, 7 stickers on a page. Josh would have (5x7)+2 = 35 stickers. Roman would have (3x7)+12 = 33 stickers. The problem says that they have the same amount, so 7 is not the solution. Pick another number to trial
- If children find it hard, work through trials with them, starting with 2, 3, 4, then 5 (the solution)
- If children find it easy, ask them to write an explanation of how they solved the problem, with mathematical examples as proof.

Solution:

There are 5 stickers on each page.

Step 5 - Clothes Shopping

Aisha went clothes shopping. She spent \pounds 99 exactly. Find 5 different ways that she could spend \pounds 99. She could buy more than one of the same piece of clothing!



- Check that your child is confident with adding more than two numbers, either mentally or using pencil and paper.
- It is a valid strategy to make random guesses at the amounts and type of clothing and find the totals.
- Ensure that children record their trials, or they will risk repeating.
- Look for patterns in the numbers, e.g. the £33 or the £11 items.
- If children find it hard, ask them to find just two or three ways to make £99
- If children find it easy, ask them to find even more ways of making £99

Solution:

£44 + £42 + £13 (Jacket, dress, skirt) £33 x 3 (3 pairs of jeans) £11 x 9 (9 vests) £44 + £44 + £11 (two jackets and a vest) £46 + £42 + £11 (Jumper, dress and a vest) £44 + £31 + £13 + £11 (Jacket, shorts, skirt, vest) £31 + £31 + £13 + £13 + £11 (two shorts, two skirts and a vest)

STRATEGY D -MAKE A LIST OR TABLE

Step 4 - Problem-Solving Penguins

Three birds have laid some eggs.

Peter Parrot



© STOPSPROBLEMSOLVING UK 2020

Peter and Olly laid 38 eggs in total. Olly and Billy laid 37 eggs in total. Peter and Billy laid 40 eggs in total.

How many eggs did each bird Lay?

- Your child will need to draw on their skills of trial-by-improvement, by making trials and adjusting
- It is possible to solve algebraically but this would be a very high expectation for this age group
- Set up a table to record your trials, for example:

Peter Olly Billy

• Encourage children to start with what they know, for example we know that Peter and Olly laid 38 eggs in total We can start thinking of pairs of numbers that make 38 and use that as our first trial. You could start with 30 and 8 as an example.

Olly	Billy
8	2
	<u>Olly</u>

- If Olly and Billy laid 37 eggs in total, we already have too many. Make another trial and start again!
- Encourage your child to work more systematically, for example ensuring that if the total is too high, they reduce their trial and keep a record in the table.
- Encourage resilience and persistence with the children
- If children find it hard, give them one solution to begin with.
- If children find it easy, create some new clues by making egg totals that are even higher, for example 3-digit numbers.

Solution:

Peter Parrot	- 21 eggs
Olly Owl	- 17 eggs
Billy Bluebird	- 19 eggs

Step 4 - Class Trip

Josh's class went on a class trip to a museum.



Tickets cost \pounds 11 per adult and \pounds 5 per child.

They spent $\pounds 93$ in total.

How many adults and children went to the theatre?

- Discuss what strategies that your child has learned that may help them with this problem, for example trial and error or making lists and tables.
- It will be helpful to make a list of multiples of 11 and multiples of 5, since the solution must be from these multiples. For example:

Teacher Tickets	Child tickets
11	5
22	10
33	15
44	20
55	25
66	30
77	35
88	40
	45
	50
	55
	60
	65
	70
	75
	80
	85
	90

- Now, you can look for solutions from these lists. You can pick pairs of numbers at random or be more systematic, for example taking £11 and subtracting from £93 to see that you would need £82 of child tickets, which is not possible.
- Encourage children to record their unsuccessful trials using jottings
- If children find it too hard, reduce the number of possibilities in the list above
- If children find it easy, ask them to find more than one solution or substitute higher numbers in the clue.

Solution:

3 adult tickets = £33 12 child tickets = £60

A second solution is possible: 1 child ticket (£5) and 8 adult tickets (£88). Sounds like a strange class trip!

STRATEGY E FIND THE PATTERN

© STOPSPROBLEMSOLVING UK 2020

Step 4 - Flower Power

There is a relationship between the numbers in each flower.

What is the missing number?



- Check that your child is ok with negative numbers, e.g. that -5 + 2 = -3. Show them with a number line if needed.
- Allow your child time to experiment with trying to find patterns between the numbers.
- Try different mathematical operations, e.g. multiplication, subtraction
- If they find is hard, replace the 30/-5/2 flower with 20/4/4
- If they find it easy, ask them to make up a similar problem for you to solve

Solution:

The total of each flower is 28. The missing number should be 10.

Step 4 - Pencil Patterns

Aisha makes a pattern out of pencils. This pattern has 2 squares and uses 7 pencils.



This pattern has 3 squares and uses 10 pencils.



Can you make or draw the next two patterns in the sequence? How many pencils do they need? How many squares do they make?

How many pencils will you need to make a track with 15 squares?

How many pencils will you need to make a track with 20 squares?

- Encourage children to use the 'act it out / make a model' strategy if needed, by making or drawing the patterns
- Children can then use the 'make a list or table strategy' to record how many pencils you need for each number of squares, for example:

pencils	squares
7	2
10	3
13	4
16	5

- The table can then be extended to find out how many pencils for a track with 15 or 20 squares
- Ask your child what patterns they notice in the numbers in the table.
- If they find it hard, support them by showing them that the table goes up in three's
- if they find it easy, investigate how we can multiply the squares by 3 and add 1 to make the number of pencils. This can help us with the harder questions without having to list all of the sequence. For example, for 15 squares, we can multiply 15 x 3 (45) and add 1 (46). You can then use this to ask how many pencils in a track with, for example, 150 squares, or 1000 squares.

Solution:

The next two in the pattern: 4 squares, 13 pencils 5 squares, 16 pencils

For 15 squares, you will need 46 pencils.

For 21 squares, you will need 61 pencils.

STRATEGY F SIMPLIFY THE PROBLEM



- A systematic approach to finding all combinations of three numbers is key here, so encourage children to learn from their skills in the systematic approach of 'trial by improvement' and the recording skills of 'making a list or table'
- This strategy encourages us to 'simplify the problem', so lets look at the 3number combinations that start with the number 1 and 2: 1-2-3, 1-2-4, 1-2-5.
- Next, let's look at combinations that start with 1 and 3: 1-3-4, 1-3-5. Make sure that children know that we do not need 'repeats', i.e. that 1-3-2 gives the same total as 1-2-3.
- Keep working through systematically, making the lists of combinations with 1 and 4 before moving onto combinations that begin with 2.
- If children find it hard, reduce the number of possibilities, e.g. pick two numbers from the numbers 1-4.
- If children find it easy, increase the number of possibilities, e.g. pick three numbers from the digits 1-7 or 4 digits from the numbers 1-9.

Solution:

3-4-5 (12)

These combinations are possible, with the totals in brackets:

1-2-3 (6) 1-3-4 (8) 1-4-5 (10)	1-2-4 (7) 1-3-5 (9)	1-2-5 (8)
2-3-4 (9) 2-4-5 (11)	2-3-5 (10)	

There are 4 ways of making an odd total from three numbers picked from the numbers 1-5.

Step 4 - Palindromic Dates



In the classroom, we might write the date like this: 06.04.2020, which is the 6th April 2020.

Luca has been thinking about "palindromic dates". A palindromic date is the same backwards as it is forwards.

An example would be 10-02-2001. Look carefully - it is the same backwards as it is forwards!

How many palindromic dates are there between 2000 and 2022?

- Make sure children are familiar with recording the date in an 8 digit format.
- It would be useful to revise the number of days in each month to be sure they know this by heart.
- Encourage children to break the problem into parts to simplify it.
- Start with the year 2000. What would the day and month be if the year was 2000: ?? ?? 2000. The date would have to be 00 02 2000. We cannot have 00 as the day, so that is not possible.
- Move on to 2001. If we use the same format: ?? ?? 2001, the date would be 10-02-2001. This is a valid date, so we have found one solution
- Work through until you have found all the solutions.
- If they find it hard, give them them the list of years with possible solutions
- If they find it easy, ask them to look for dates outside of the 21st century and see how many they can find.
- Another challenge would be to look at palindromic dates in a 6-digit format, e.g. 02.11.20.

Solutions:

10-02-2001 20-03-2002 01-02-2010 11-02-2011 21-02-2012 02-02-2020 12-02-2021 22-02-2022

Step 5 - Out for dinner

A restaurant has 20 square tables.

One table can sit 4 people.

If two tables are joined together, they can seat 6 people.

If three tables are joined together, they can seat 8 people.

Tonight, they need to seat 62 people in total.

How can they use their 40 tables to seat 62 people?



• It is very useful to sketch this problem out:



• Help your child to draw on their skills learned in strategy D - "Make a List Or Table." The information could be recorded in a table:

Tables	Number of seats
1	4
2	6
3	8

- Using this list or by drawing, we can see that one long line of 20 tables would only sit 42 people.
- 20 individual tables would sit 80 people. So we need a combination of groups of tables.
- Encourage children to experiment with groups of tables to see if we can seat exactly 62 people.
- If children find it hard, give them the clue that we are using a certain number of individual tables.
- If children find it easy, ask them to find more than one solution

Solution:

10 individual tables would seat 40 people. A line of 10 tables would seat 22 people, giving the total of 62 people using 20 tables.

Step 5 - Horse Race



Aisha takes her horse to a horse race.

Altogether, counting people and horses, there are 80 eyes and 130 legs.

How many horses and people are there?

- Children must be able to mentally multiply by 4. Doubling and doubling again is a good mental strategy for this.
- The strategy is to simplify the problem, so look for clues that are easiest to solve. We know that they all have two eyes, so if there are 80 eyes, there must be 40 horses and people in total.
- We can now just make trials to guess at at amount of horses and humans that totals 40, for example, 30 horses and 10 humans.
- We can easily calculate how many legs are on display if there are 30 horses and 10 humans, recording the results in a table.

Horses	Horses's legs	Humans	Human's legs	Total Legs
	-		-	
30	120	10	20	140

- There are too many legs here, so we can try another, e.g. 20 humans, 20 horses and calculate.
- If children find it hard, pick their trials for them and model how to calculate the numbers of legs by multiplication.
- If children find it easy, ask them to set a number of horses and humans for you to solve.

Solutions:

- 25 horses and 15 humans.
- 25 horses = 100 legs; 15 humans = 30 legs. 130 legs in total.
- 40 creatures in total, making 80 eyes.

© STOPSPROBLEMSOLVING UK 2020

STRATEGY G WORKING BACKWARDS

Step 5 - Party Time!

A)

At the party, half of the guests left at spm. Half of the remaining guests left at 6pm. The last 12 guests stayed until 7pm. How many guests were at the party?



B)

Every 2 minutes, 6 people started dancing. How many minutes did it take for all of the guests to be dancing?

- Check that your child is familiar and confident with 'inverse operations', e.g. that multiplication is the inverse of division.
- You could start by asking them: ? ÷ 3 = 12 and discussing how to calculate it using multiplication.
- This strategy encourages children to work backwards so model the problem by working backwards through the clues. You could use arrows as a flow diagram, e.g.:



- Ask the children what number could be in the middle question mark, representing 6pm. It would be 12 x 2 = 24
- Repeat the process for the next step and we establish that there must have been 48 people at the party at the beginning.
- For part B, set up a simple table:

Minutes	People dancing
0	0
2	6
4	12

Solutions:

There were 48 guests at the party

It took 14 minutes for all 48 people to be dancing.

Step 5 - Josh's Cousins

Josh has cousins that were born at intervals of 3 years between each one.

The oldest is now exactly 3 times older than the youngest.



How old is the middle cousin of the 5?

- A year 4 child should be confident to count up in 3's.
- Trial-by-improvement would be a valid strategy here, for example by guessing a starting age and working through the 5 cousins to see what age the oldest would be.
- To work backwards, we can see that there must be 12 years between the youngest and the oldest:

```
Cousin 1
+3 years
Cousin 2
+ 3 years
Cousin 3
+ 3 years
Cousin 4
+ 3 years
Cousin 5
```

• Continuing the 'work backwards" theme, we can now look at ages for the youngest and oldest that are 12 years apart, looking for two ages where one is exactly 3 times higher than the other, for example 1 & 13, 2 & 14, 3 & 15 etc.

Solution:

The youngest is 6, the oldest is 18. This means that the cousin in the middle is 12 years old.

STRATEGY H -SOLVE ALGEBRAICALLY

Step 4 - Algebra of Ages



A)

Josh is 3 times older than his sister. She is 8 years younger than he is. How old is Josh?

B)

Josh's cousin is 5 times older than he was 20 years ago. How old is Josh's cousin?

- While these problems can be 'purely' solved algebraically, it is most useful at year 4 to show your child how to model these in terms of algebra, then use other strategies to solve.
- Part A can be modelled as: Josh' age = sister's age x 3, or J = 3 x s
- These can then be put into a table and solved using trial-by-improvement, e.g.:

<u>Sisters age (s)</u>	Josh's age (3 x s)
1	3
2	6
3	9
4	12
5	15
6	18

- We can then look for the only pair of ages that have a difference of 8: 4 and 12. This can be expressed in terms of algebra also: Josh's age = sister's age + 8, or J = s + 8.
- At year 4, just expressing real-life situations in terms of algebra is a high expectation and not all children will understand.
- Part B can be approached in a similar way and expressed algebraically before using a table.
- We could express the situation as: Cousin's age \div 5 = cousins age 20.
- We can then "substitute" ages into the expression. A table could be used to record the trials:

cousin's age	cousin's age ÷ 5	<u>cousins age - 20</u>
40	8	20
30	6	10
20	4	0
25	5	5

Solutions:

- A: Josh is 12 and his sister is 4
- B: His cousin is 25 years old.

Step 4 - Shack time

Josh buys some chips and a drink on his way home.

One drink and a portion of chips costs $\pounds 1.80$.

A drink costs exactly half the cost of a portion of chips.

How much does the drink cost? How much do the chips cost?

How much would Josh spend on two drinks and a portion of chips?



- Again, helping children to express this situation using letters or symbols is all that is expected at this level.
- For example, we could express the costs as:

chip cost = drink cost x 2 or c = 2 x d or c = d + d

- We could also say: drink + chips = £1.80, or d + c = £1.80
- From here, we can substitute values for the drink and chip costs using the 'trial by improvement' and the 'make a list or table' strategy. We can pick a random trial first of all:

cost of chips	cost of drink	total cost
£1.00	50n	£1.50
£2.00	£1.00	£3.00

- you can keep making and adjusting trials until the correct answer is found.
- if children find this easy, you could show them that if we know c = d + d and that d + c = £1.80, then d + d + d = £1.80. We can simply divide £1.80 by 3 to find the cost of s a drink.
- if children find this difficult, then ignore the algebraic notation and use symbols to represent a drink or chips, then solve by trial and error.

Solutions :

The chips cost £1.20, the drink costs 60p.

Well done - you have solved all of the year 4 logic and reasoning problems in this booklet and you are an expert problem-solver!

Head over to <u>www.stopsproblemsolving.co.uk</u> and check out the COVID19 zone for more games and problems.